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MARINE METEOROLOGY

Growth of Rain in Warm Clouds

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Director

Preface

During the summer of 1953, Dr. Raymond Wexler carried on his research in cloud physics at this Institution, as a part of the program in marine meteorology. His study of warm rain from trade-wind cumulus clouds near Hawaii, which was carried out during this period, appears herein. Much of the data used in this study was kindly provided by Mr. Alfred Woodcock and Mr. Duncan Blanchard and many helpful comments were provided by them along the way.

Joanne S. Malkus

Abstract

Observations and theory of rain from warm clouds are reviewed. An analysis of the role of giant sea nuclei in these clouds indicates that there is a critical nucleus size which can become the largest raindrops, while larger nuclei become smaller raindrops.

A theoretical analysis is made of the production of rain in warm clouds. The theory is applied to the mean data taken by Blanchard in orographic rain in Hawaii. Mean liquid water contents of about 1 g m^{-3} and updraft velocities of about 1 m sec^{-1} in light rain and about 3 m sec^{-1} in heavy rain are found.

I. INTRODUCTION

In previous studies, measurements of temperature, water vapor content, and turbulence in trade wind cumulus clouds, made from slow flying aircraft, were analyzed, (Malkus 1954). Using a theoretical steady-state entrainment model, numerous calculations of entrainment, liquid water content and updraft velocities were made from the observations. Mean liquid water contents in trade wind cumulus were found to be considerably less than that derived from the adiabatic ascent of a parcel of air. In general, updraft velocities were found to increase with height above the cloud base, attaining values of a few m/sec in the upper portions of the cloud. More recently, the data has been analyzed to determine the life history of large convective bubbles of rising air in cumulus clouds (Malkus and Scorer 1954). Indications were found that the bubbles erode at a constant rate as they ascend in the cloud. The interaction of bubbles to form large cumulus clouds was discussed.

In these studies no observations or analyses were made of clouds producing rain, which frequently occur in trade wind cumulus of appropriate depth. Temperatures throughout many of these clouds are entirely above 0°C (referred to hereafter as warm clouds). It is the purpose of this paper to discuss some aspects of the production of rain from warm clouds, and to present a method of analysis of rainfall data measured at different levels in these clouds. By means of such an analysis it will be possible to determine other properties of the cloud, such as liquid water content and updraft velocity.

II. OBSERVATIONS AND THEORY OF WARM RAIN*

Previous to World War II it was believed by many meteorologists that only the ice phase could initiate precipitation in clouds, but many observations, both from temperate and tropical latitudes, show conclusively that rain may occur from clouds entirely above freezing. Rain from warm clouds has so far been observed over or near oceans and generally in tropical regions. In the Bahamas warm rain falls frequently from a combination of stratocumulus and cumulus clouds (Virgo 1950). In the trades near Guam, moderate rain showers have fallen from warm stratiform decks 4,000 feet thick. A survey of warm rain from orographic clouds in Hawaii (Mordy and Eber 1954) reveal that the thickness of such clouds are generally 5,000 feet or more, although on one occasion appreciable rain fell from a cloud less than 4,000 feet thick. During a 10-day period of observation, during which appreciable rain fell, the clouds were cumuliform with base at 2,000 feet above sea level and were capped by a thin stratiform layer, a few hundred feet thick, at the base of the inversion at an altitude of 7,000 feet. In Australia observations of warm rain clouds from aircraft and by radar show that in 4 cases the depths of the cumulus clouds producing heavy rain intensities (15 to 70 mm/hr) were between 9,000 and 10,000 feet; there was one case of light rain from a cloud 6500 feet deep (Styles and Campbell 1953). In general the duration of the rain was only about 15 minutes. Observations of trade wind cumulus indicate that appreciable wind shear within the cloud inhibits the production of rain.

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* Some of the information in this section is included in a chapter by the author in a forthcoming book: Tropical Meteorology by H. Riehl.

Radar observations in Ohio show that the first radar echo from developing showers in cumulus clouds occur frequently at temperatures above 0°C. Subsequently the echo elongates, both upwards and downwards, and soon extends to heights where the temperature is considerably below 0°C. Although the cloud tops may be several thousand feet above the top of the initial radar echo, nevertheless the interpretation is made that warm rain occurs initially in these clouds (Battan 1953). These observations are the first indication that warm rain may occur over continental interiors.

Although Findeisen (1939) rejected as contrary to experience the theory that cloud drops can grow by collision with smaller drops to raindrop size, nevertheless his calculations showed that a drop 1.4 mm in diameter can grow by this process in a cloud 2,000 meters thick. The theory was revived by Langmuir (1948) who showed that large drops will overtake and collide with smaller droplets in its path, due to differences in fall velocity. Because of the aerodynamic flow pattern, some of the smaller drops would flow around the large drop without being caught. He introduced the concept of collection efficiency E, defined as that fraction of the liquid water in its geometric path caught by the larger drop. A theory was developed for computing E, which was later found to be valid for large drops (Gunn and Hitschfeld 1951), but his values for drops smaller than 0.1 mm in diameter are probably invalid since he assumed the small droplets to have negligible dimensions. Langmuir calculated the time required for drops of various sizes to grow to a diameter of 6 mm, approximately the size at which falling raindrops break up into smaller ones, and reasoned that a chain reaction of drop growth would occur thereafter. However, drop size measurements in warm rain indicate that the drops do not reach break-up size (Blanchard 1953).

Bowen (1950) computed the growth of a drop 25μ in diameter in a cloud containing 20μ drops and a liquid water content of 1 g/m^3. Using Langmuir's collection efficiencies he computed time-height trajectories of the large drop for various updraft velocities. He showed that the final drop size depends primarily on the updraft velocity and is independent of the assumed values of the collection efficiency or liquid water content. His results show that in a cloud with an updraft of 1 m/sec, a drop is carried aloft about 7,000 feet and then descends, attaining a diameter of 1.5 mm at the cloud base.

A basic difficulty in the theory of the growth of drops by coalescence is the initial slow rate of growth. In a cloud containing 1 g/m^3 of liquid water, a drop initially 20μ in diameter would grow to 40μ in about one hour, even if a relatively high value of the collection efficiency were assumed. During this period in cumulus clouds the droplet would be carried by the updraft to the cloud top where it would be unable to resist evaporation and to descend through the updraft. A solution to this difficulty was suggested by Ludlam (1951), who postulated that large drops 40 to 80μ in diameter are already present near the cloud base. He attributed their origin to the giant sea salt nuclei found by Woodcock (1952) to exist in the atmosphere over or near the sea at altitudes up to 10,000 feet. When brought into clouds these nuclei can grow by direct condensation to diameters of 40 to 80μ within several hundred feet above the cloud base. Ludlam computed the subsequent growth of these drops in a cloud in which the liquid water content was governed by the adiabatic ascent of a parcel of air. His calculations indicated that if rain is to be initiated in a cumulus cloud, a drop must attain a diameter of 0.3 mm by the time it reaches the cloud top.

This size is sufficient for the drop to resist evaporation during a fall of several hundred meters from cumulus peaks and to settle back into the cloud bulk and resume growth. His results indicate that for an updraft of 2m/sec a minimum cloud depth of 4500 to 6,000 feet is required for the formation of rain.

Measurements by Weickmann and AufmKampe (1953) reveal that the drop diameter in small cumulus clouds ranges from 6 to 60 μ whereas in cumulus congestus and cumulonimbus it extends to 0.2 mm or more. An analysis of their data by East and Marshall (1954) indicates that turbulence in cumulus clouds increases the collection efficiency of small cloud drops thereby facilitating their growth to raindrops.

The growth of drizzle in stratiform clouds less than 2,000 feet thick has been analyzed by Mason (1952). He considered that turbulent diffusion limits the life of droplets in clouds and showed from probability theory that a few drops are likely to remain within the cloud for a period of a few hours, during which they may grow by condensation and by coalescence to a drop diameter of 0.2 to 0.3 mm.

In conclusion, although it has been shown theoretically that giant sea salt nuclei are sufficient to initiate rain in warm clouds, it is not known whether they are a necessary condition for its formation. It is doubtful whether these nuclei are present over continental interiors, such as Ohio, where radar evidence indicates the presence of warm rain initially in cumuliform clouds. Many of the direct observations of warm rain have been from clouds with some stratiform characteristics. If the drops remain within such clouds for a period of the order of an hour, they may attain the size of a small raindrop without the aid of giant salt nuclei.

III. GROWTH OF SEA SALT NUCLEI IN WARM CLOUDS

The growth of sea salt nuclei by condensation for various relative humidities and temperatures have been analyzed theoretically and experimentally by Keith and Arons (1953). From their work it may be shown that drops containing sea salt nuclei of mass 100$\mu\mu$g and 1,000$\mu\mu$g are likely to have respective diameters of 20μ and 40μ at the base of a cloud. Their further growth within the cloud by condensation and by coalescence may also be computed provided that suitable values of the collection efficiency may be assumed. According to Langmuir (1948) drops smaller that 30μ have zero collection efficiency and therefore must grow entirely by condensation. Drops with diameter greater than 0.1 mm have collection efficiencies (for aerodynamic flow) approaching 0.9; these drops grow mainly by coalescence. For the purpose of estimating the action of sea salt nuclei in warm clouds, let us assume a model cloud 5,000 feet thick with a mean liquid water content of 1 g/m^3 and an updraft velocity of 60 cm/sec. As will be shown later from the Hawaiian data, light rain may be expected to fall from such a cloud. For the computations the following collection efficiencies are assumed:

Table I

Assumed values of collection efficiencies

Diameter (μ)	E (%)
<30	0
30-60	30
60-100	60
>100	90

With the aid of the graphs of Keith and Arons and equation (1) (below) and Table I, it may be found that the 20 μ drop containing the 100 $\mu\mu$g salt nucleus, is carried aloft to a height of 5900 feet in the model cloud, at which level it has acquired a fall velocity equal to the updraft velocity. Thereafter it descends through the cloud and attains a raindrop diameter of 0.94 mm at the cloud base; the total time for the trajectory is 50 minutes. On the other hand the 40 μ drop, containing the 1,000 $\mu\mu$g nucleus, is carried up to a height of 4700 feet and then descends to the cloud base with a diameter of 0.63 mm; the total time for this trajectory is 30 minutes.

It is apparent that drops with nuclei larger than 1,000 $\mu\mu$g will become raindrops smaller than 0.6 mm. Nuclei smaller than 100 $\mu\mu$g will be carried to the top of the cloud without attaining a fall velocity equal to the updraft. In cumuliform clouds such droplets will evaporate into the cloudless air above. If, however, there is a stratiform layer caused by the spreading out of cumulus clouds at the base of an inversion, such as exists frequently in Hawaii, a few of these droplets may remain in this layer sufficiently long to attain a size and fall velocity to descend through the cloud bulk and attain maximum raindrop size.

Although it has been demonstrated by Woodcock that both raindrops and sea salt nuclei have size distributions showing an exponential decrease in number with increasing size, it cannot be concluded that there is a one to one correspondence between nuclei and raindrops, such that large nuclei become large raindrops and small nuclei become small raindrops. The sample analysis above indicates that for nuclei introduced near the base of a cloud there exists a critical size which may reach maximum raindrop size while larger nuclei become smaller raindrops. The reason that there are so few large raindrops is to be found in the time of growth. A large nucleus attains the size of a small raindrop in a relatively short time. Because of the much longer times of growth for the smaller nuclei, many may never reach raindrop size because of evaporation at the cloud boundaries during changing cloud conditions.

IV. THEORY OF GROWTH BY COALESCENCE

After a droplet has reached a diameter of about 40 μ, its subsequent growth in a water cloud is almost entirely by coalescence. In a steady rain it may be assumed that the difference in rain intensity or maximum drop size at two levels in a cloud is derived from the growth of the drops by coalescence with cloud liquid water between those levels. In this section a theoretical analysis is made of the information that may be obtained from measurements of the rain intensity and maximum drop size at different levels of a cloud from which steady rain is falling.

The growth of raindrops of mass m and diameter D by coalescence with smaller cloud drops is given by

$$\frac{dm}{dt} = \frac{\pi D^2 EVL}{4} \qquad (1)$$

where E is the efficiency of catch, V is the fall velocity of the raindrops relative to the cloud drops, and L is the cloud liquid water content. From (1) we obtain

$$\frac{dD}{dt} = \frac{EVL}{2} \qquad (2)$$

If w is the updraft velocity and z the height, (2) may be written:

$$\frac{dD}{Dz} = \frac{EVL}{2(V - w)} \qquad (3)$$

If drops of diameter D_1 grow to drops of diameter D_2 over a height interval Δz, then

$$D_2 - D_1 = \frac{KL\Delta z}{2} + w\int_{D_1}^{D_2} \frac{dD}{V} \qquad (4)$$

where L and w now represent mean values over the given height interval. The integral on the right side represents the time for a drop to grow from D_1 to D_2 in a cloud of effective liquid water content $KL = 2\ g\ m^{-3}$ and is plotted in fig. 1.

The variation of liquid water content with height may be determined from an equation of continuity representing cloud production by the updraft, advection, the storage and precipitation of liquid water:

$$\frac{d}{dt}\left(q + \frac{L}{\rho}\right) = \frac{1}{\rho}\frac{\partial R}{\partial z} \qquad (5)$$

where q is the specific humidity, ρ is the air density, and R is the rain intensity. Hence

$$\frac{\partial}{\partial t}\left(q + \frac{L}{\rho}\right) + u\frac{\partial}{\partial x}\left(q + \frac{L}{\rho}\right) + v\frac{\partial}{\partial y}\left(q + \frac{L}{\rho}\right) + w\frac{\partial}{\partial z}\left(q + \frac{L}{\rho}\right) = \frac{1}{\rho}\frac{\partial R}{\partial z} \qquad (6)$$

It will be assumed that the change of q and L with time (first term on left hand side) are comparatively small. Horizontal advection (second and third terms) of the specific humidity may also be neglected, but horizontal advection of L may be important. This latter term will be included only implicitly by modifying the term $w\frac{\partial q}{\partial z}$ which represents the rate at which liquid water is being created by the updraft. Equation (6) then becomes:

$$fw\frac{\partial q}{\partial z} + w\frac{\partial}{\partial z}\left(\frac{L}{\rho}\right) = \frac{1}{\rho}\frac{\partial R}{\partial z} \qquad (7)$$

where f may be defined as the percentage effectiveness of the updraft in

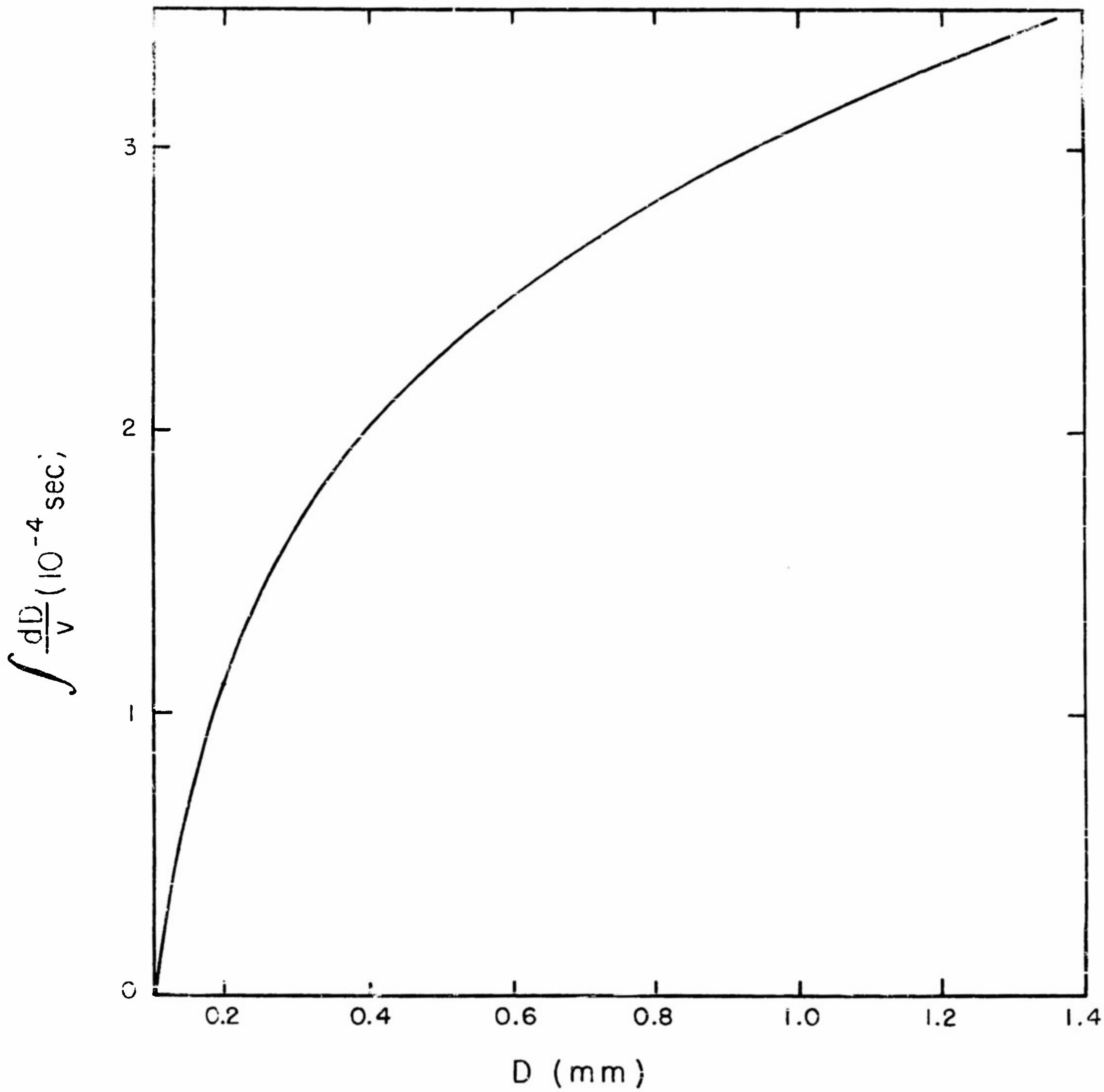

FIG. 1

creating liquid water. For a parcel ascending adiabatically $f = 1$. The portion $1 - f$ may be considered to be lost to the cloud due to horizontal mixing or entrainment. The liquid water content of a parcel of air ascending adiabatically may be derived from (7) by setting $R = 0$ and $f = 1$.

We may define the efficiency of rain production by a cloud as the ratio of the rain intensity at the cloud base to the amount of condensation made available by the updraft:

$$F = \frac{R}{\rho w \Delta q} \tag{8}$$

where Δq represents the difference in specific humidity between the cloud base and top.

The rain intensity is defined by:

$$R = \sum nm(V - w) \tag{9}$$

where n is the number of drops of mass m. From continuity in steady rain the number of drops entering a given volume is equal to the number of drops leaving:

$$n(V - w) = \text{constant} \tag{10}$$

provided no drops originate or evaporate within the volume. Adderley used the equation: nV = constant for drops greater than a given size, and concluded from his data that the efficiency of catch of large drops, greater than about 1 mm in diameter, is about four times that of drops smaller than 0.5 mm. This conclusion is contrary to experimental and theoretical evidence. Adderley neglected the updraft velocity which affects the small drops more than the large ones. Furthermore the time spent within a given height interval is considerably greater for the small drops, so that steady conditions, required for the use of (10), are more unlikely.

V. THE HAWAIIAN DATA

In this section the theory will be applied to the data for orographic rain in Hawaii (Blanchard 1953). Much of this data was taken at two stations on the northeast slope of the island of Hawaii: Station 4 at 3,000 ft msl and Station 5 at 5,500 ft msl. The stations are located along the slope leading to the saddle between two mountain peaks: Mauna Loa and Mauna Kea, both extending to heights near 14,000 ft. A schematic diagram of distance, wind and cloud conditions in orographic rain is shown in fig. 2.*

In order to apply the theory to the data it is assumed that the mean rainfall at the higher elevation (Station 5) is representative of the mean rainfall at the same elevation in the free atmosphere from which the rain at the lower elevation (Station 4) is derived. This assumption is admittedly difficult to justify for individual observations since, due to the motion of the air upslope, the rain would in general occur at an earlier time than at the higher elevation. Moreover, the salt nuclei that initiate the rainfall may be quite different at the two locations. However, the assumption may be valid for mean rainfall data because the cloud depth decreases uniformly up the slope due to the constant level of the inversion on the windward side of the slope. Since most of the growth of the large raindrops occurs during their descent through the cloud, the mean maximum drop size and the rainfall intensity should decrease with height in the cloud in approximately the same manner as it decreases along the mountain slope. The proportionality of rainfall amounts to cloud depths is indicated by annual rainfall maps of the region which show a maximum at Station 4 and a uniform decrease of rainfall amounts upward along the slope.

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* Thanks are due Mr. A. H. Woodcock of the Woods Hole Oceanographic Institution for suggesting this diagram.

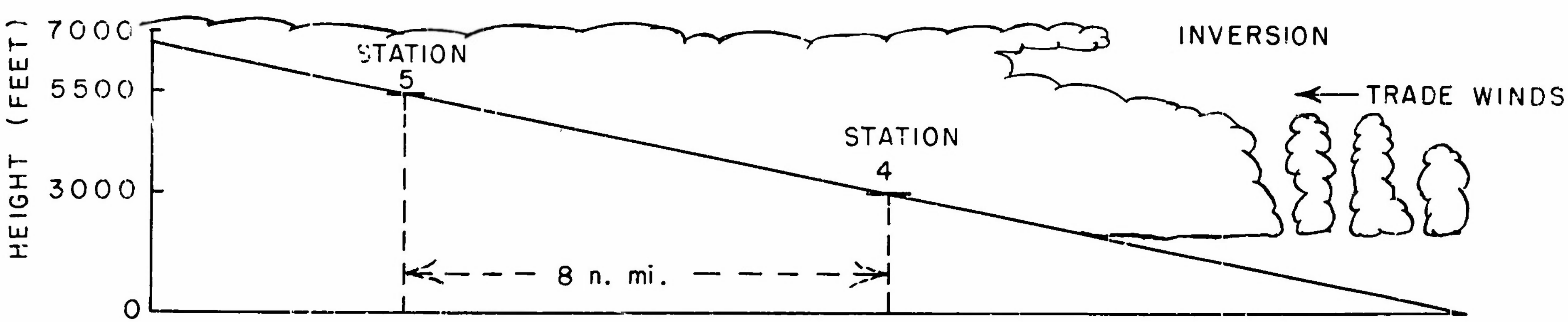


FIG. 2 SCHEMATIC DIAGRAM OF CLOUD FORMS IN OROGRAPHIC RAIN ON THE ISLAND OF HAWAII.

The data for orographic rain were taken on four separate days during 1952: for Station 4, May 6 and July 8 (after 1714 Hawaiian time), and for Station 5, April 28 and 29. On these days the height of the inversion, as revealed by the Hilo radiosonde, was between 7,000 and 8,000 feet. Below the inversion the temperature followed approximately the moist adiabatic lapse rate with a mean of 15C at 3,000 feet and 11C at 5,500 feet. The mean temperatures on May 6 and July 8 were about 1°C higher than on April 28 and 29, but the effect of these differences on the rainfall is considered to be small.

The data for orographic rain appear to be divided somewhat naturally into two categories: light rain and heavy rain. For Station 5 the data for light rain range in intensity from 0.05 to 0.7 mm hr^{-1}, and heavy rain 1.1 to 2.5 mm hr^{-1}. For Station 4 light rain ranged from 0.5 to 3 mm hr^{-1} and heavy rain 3.6 to 13.0 mm hr^{-1}. Table II shows the mean number of drops arriving on a horizontal area of 1 m^2 in 1 second for each drop size interval. The mean rain intensities, as computed from the drop size data, are included.

Table II

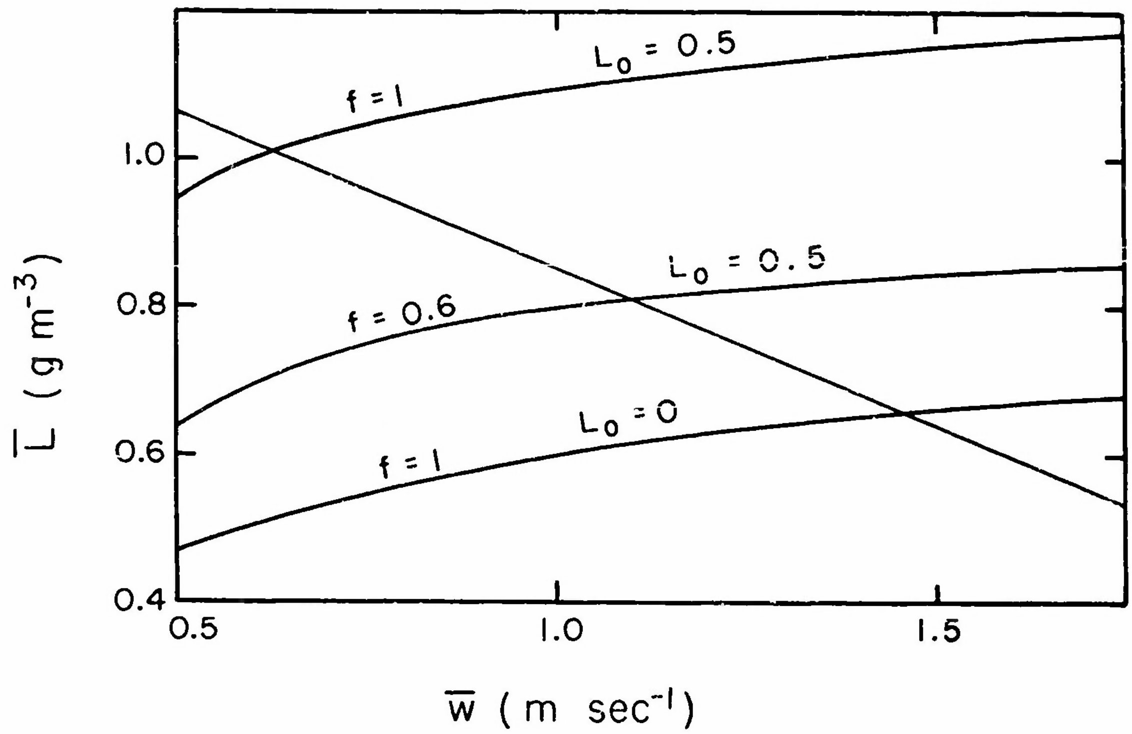
Mean number of drops ($m^{-2}sec^{-1}$) and rain intensities (mg m^{-2} sec^{-1}) at Station 4 (3,000 feet) and Station 5 (5,500 feet). (Computed from Blanchard 1953).

Drop diam. (mm):	0.1	0.3	0.5	0.7	0.9	1.1	1.3	Rain Int.	No. of cases
			Light rain						
Station 5:	15,100	3160	105					100	13
Station 4:	990	4760	1920	650	157			420	7
			Heavy rain						
Station 5:	10,050	11,350	2420	158	15			450	6
Station 4:	3,830	6,560	4070	2110	1060	425	103	1670	6

It is assumed from the data for light rain that raindrops 0.5 mm in diameter at 5,500 feet grow by coalescence to 0.94 mm at 3,000 feet and that the respective rain intensities at the two levels are 100 and 420 mg $m^{-2}sec^{-1}$. It is likewise assumed for heavy rain that 0.9 mm drops at 5,500 feet grow to 1.35 mm at 3,000 feet with respective rain intensities of 450 and 1670 mg $m^{-2}sec^{-1}$.

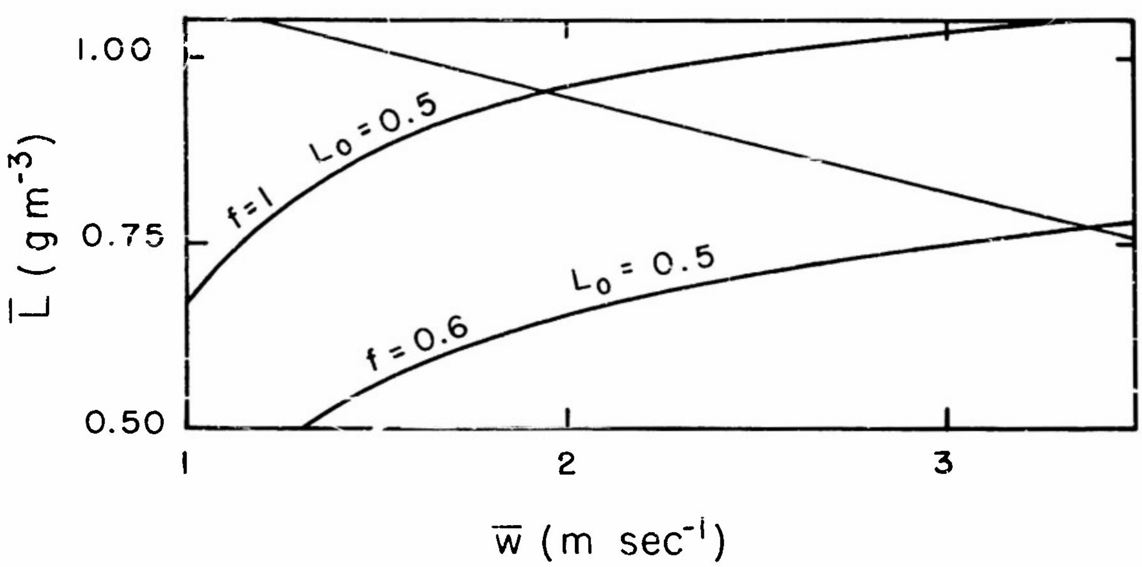
Fig. 3 illustrates the method by which the liquid water contents and updraft velocities are derived from the above data. The straight line represents the solution of (4) for drop growth, while the curved lines represent the solution of (7) using rain intensity differences. A linear increase in the liquid water content between 3,000 and 5,500 feet was assumed for the solution of (7). For light rain with adiabatic conditions ($f = 1$) and a liquid water content of $L_0 = 0.5$ g m^{-3} at 3,000 feet, it is seen from the diagram that a mean updraft velocity of 60 cm sec^{-1} and a mean liquid water content of 1.0 g m^{-3} satisfies the data. If the liquid water content at 3,000 feet were $L_0 = 0$, then $w = 1.5$ m sec^{-1} and $L = 0.66$ g m^{-3} would satisfy the data. In view of the fact that the cloud base was frequently at an elevation of 2,000 feet a value of $L_0 = 0.5$ g m^{-3} at 3,000 feet appears more appropriate. For non-adiabatic conditions, where $f = 0.6$, then $w = 1.1$ m sec^{-1} and $L = 0.80$ g m^{-3} satisfies the data. There is no independent way of estimating the value of f, so that the solution may only be derived within certain limits.

Turning to the heavy rain case (fig. 4) it is found that a mean liquid water content of 0.95 g m^{-3} and a mean updraft velocity of 2 m sec^{-1} satisfies the data for adiabatic conditions. For non-adiabatic conditions with $f = 0.6$, mean values of 3.4 m sec^{-1} and 0.77 g m^{-3} are found.



$L_0 = 0.5$
$f = 1$
$L_0 = 0.5$
$f = 0.6$
$L_0 = 0$
$f = 1$
1.0
0.8
0.6
0.4
0.5
1.0
1.5
$\bar{L}$ (g m^{-3})
$\bar{w}$ (m sec^{-1})

FIG. 3



1.00
0.75
0.50
$f=1$
$L_0 = 0.5$
$L_0 = 0.5$
$f = 0.6$
1
2
3
$\bar{L}$ (g m^{-3})
$\bar{w}$ (m sec^{-1})

FIG. 4

In the upper portion of the cloud there are two more unknowns: the height of the cloud top and the maximum drop size at the cloud top. Further assumptions are therefore necessary in order to determine an approximate solution. The assumption will be made that the terminal velocity of the maximum drop size at the cloud top is equal to the average updraft velocity between the 5,500 foot level and the cloud top. It is also necessary to estimate the mean liquid water content of the top portion of the cloud by means of continuity from the lower portion. For the light rain case where $L_0 = 0.5$ g m^{-3} and $f = 1$, we know from fig. 3 that the liquid water content at 5,500 feet is about 1.5 g m^{-3}. We therefore estimate that the mean liquid water content from that level to the cloud top is between 0.9 and 1.2 g m^{-3}. Using these values as limits it is found by the use of equation (4) that the cloud top is between 7,100 and 6,600 feet for a mean updraft velocity of 50 cm sec^{-1}. Higher updraft velocities would mean a lower cloud top. Since the height of the inversion was observed to be between 7,000 and 7,500 feet, the above solution appears reasonable for the light rain case.

For heavy rain it is estimated in a similar manner that the mean liquid water content in the upper portion of the cloud is about 1 g m^{-3}. From (3) it is found that a mean updraft velocity of 1.5 m sec^{-1} gives a cloud top of 7,100 feet, while a 1.0 m sec^{-1} updraft gives 7,900 feet.

The only information that equation (7) gives us for the upper portion of the cloud is that most of the liquid water produced by the updraft in this region is wasted away into the environment. Much of the growth that occurs in this region is at the expense of liquid water carried up from the lower portion of the cloud. We may estimate the efficiency of rain production for the above solutions by applying (8). For light rain an efficiency of 29% is found; for heavy rain it is 34%. For thunderstorms an efficiency of rain production of about 20% was found by Braham (1952).

VI. THE AUSTRALIAN DATA

Because of the large interval between measured drop sizes in the Australian data, the computed rain intensities at different levels are subject to considerable error. Due to such errors, the results of applying (4) and (7) to the data are frequently either unreasonable or inconsistent with other observations. In particular the computed mean liquid water content in the lower portion of the cloud is often found to be greater than the maximum possible mean liquid water content as derived from the parcel method. For this reason, the measurements for only one case from the Australian data will be analyzed here: March 18, 1950 as listed in Adderley's Table 3. Although this case is not completely consistent with observation, the results will illustrate the type of problem that arises in the analysis of the data. In the following table are shown the rain intensities at different levels and the drop diameters for $n(V - w) = 25\ m^{-2}\ sec^{-1}$, where n refers to the number of drops greater than the indicated diameter.

Table III

Rain intensities and drop diameters (Adderley's Table 3).
The OC level is at 13,000 feet.

Height (feet)	Rain intensity ($mg\ m^{-2} sec^{-1}$)	Drop diameter (mm)
9,000	225	1.30
10,000	140	1.13
11,000	94	0.93
12,000	65	0.78
13,000	56	0.70

Applying (4) and (7) to this data, it is found that the following distribution of liquid water and updraft velocity represent a solution:

Table IV

Mean liquid water contents and updraft velocities.

Height interval (thousand feet)	Liquid water ($g\ m^{-3}$)	Updraft ($m\ sec^{-1}$)	f (%)
9-10	0.74	2.0	100
10-11	1.10	1.0	45
11-12	1.00	0.5	0
12-13	0.48	0.5	0

According to Adderley the base of the cloud was between 9,000 and 10,000 feet. For any reasonable updraft velocity, the drop size data require a liquid water content greater than would be derived for a parcel of air ascending adiabatically from 9,000 feet. If a liquid water content of 0.4 $g\ m^{-3}$ at 9,000 feet were assumed, implying a cloud base below 9,000 feet, then the values derived in Table IV are consistent with the drop size data. A mean updraft velocity of 2 $m\ sec^{-1}$ in the lowest portion of a stratiform cloud deck appears somewhat high, but appreciably lower updraft velocities would require impossible values of the liquid water content.

Between 10,000 and 11,000 feet somewhat different values of L, w and f are possible for consistency with the data. However, appreciably lower values of the updraft would make it discontinuous with the derived values in the lower portion of the cloud, and appreciably higher updrafts would require sharply lower values of f. Between 11,000 and 13,000 feet the drop growth can be explained almost entirely by liquid water being carried aloft from lower elevations, an indication of considerable evaporation of the cloudy air into the environment at the upper levels of the cloud. The values derived in Table IV indicate that only about 11% of the water produced by the updraft precipitates from the cloud.

VII. SUMMARY

Although direct observations of warm rain have so far been made over or near the sea, interpretations of radar observations indicate that warm rain also occurs over continental interiors at least initially in showers. More direct observations of the phenomenon over inland areas are desirable. Theory indicates that giant salt nuclei near the base of a cloud can initiate warm rain in cumuliform clouds. In the absence of these nuclei, cloud drops may grow to raindrop size if, perhaps with the aid of turbulent diffusion, they remain within the cloud for a period of about an hour.

On the assumption that giant salt nuclei are introduced into the base of a warm cloud, it was shown that there exists a critical nucleus size which can reach maximum raindrop size, while larger nuclei become smaller drops and smaller nuclei, in general, do not attain raindrop size.

A theoretical analysis was made of the information that may be obtained from rainfall data at different levels in warm clouds. If it is assumed that the differences in rain intensity and maximum drop size at the two levels are due to growth by coalescence between those levels, the mean liquid water contents and updraft velocities between those levels may be computed, provided that suitable estimates may be made of the entrainment of cloud free air. The theory, applied to the mean rainfall data for orographic rain measured at two different levels along the slope of the island of Hawaii, indicates a mean liquid water content of about 1 g m^{-3} throughout the cloud. Mean updraft velocities in the lower 3,000 feet of the cloud range from about 0.6 to 1 m sec^{-1} for light rain, and 2 to 3.5 m sec^{-1} for heavy rain. About 30% of the liquid water made available by the updraft falls from the cloud as rain, while the remaining 70% is wasted away into the environment mainly from the upper portions of the cloud.

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Titles for Illustrations

Fig. 1. Value of integral $\int_{D_1}^{D_r} \frac{dD}{V}$ for $D_1 = 0.1$ mm. The ordinate represents the time in hundreds of seconds for a drop to grow by coalescence from an initial diameter of 0.1 mm. in a cloud of effective liquid water content 2 g m^{-3}.

Fig. 2. Schematic diagram of cloud forms in orographic rain on the island of Hawaii.

Fig. 3. Light rain, Hawaii: Mean liquid water contents and updraft velocities required for observed drop growth (straight line) and observed increase in rain intensity (curved lines) from the 5,500 foot level to the 3,000 foot level. Values of L_o represent the liquid water content in g m^{-3} assumed for the 3,000 foot level: f is a measure of the amount of entrainment and equals unity for adiabatic condition.

Fig. 4. Heavy rain, Hawaii: Mean liquid water contents and updraft velocities required for observed drop growth and increase in rain intensity between 5,500 and 3,000 feet.

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